SIMPLE METHOD OF CALCULATING HEAT TRANSFER AND FRICTION FORCES IN A TURBULENT BOUNDARY LAYER FOR VARIABLE CONDITIONS AT THE WALL

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The concept of wall influence zones in a turbulent boundary layer on a plate is introduced. Using these zones, the formulas obtained for calculating heat transfer and friction forces for steady conditions at the wall are extended to include variable conditions.

A large number of theoretical and experimental investigations have been performed to study turbulent boundary layers on plates for steady conditions over the entire length of the plate. Simple expressions,

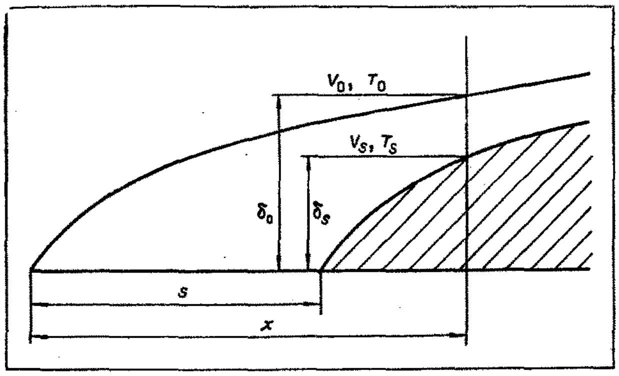

Fig. 1. Schematic drawing of the position of the influence zone at the wall.

convenient for use in engineering computations, have been obtained for the heat flow, shear stress, and boundary-layer thickness for a gas with constant physical parameters in the boundary layer. For Reynolds numbers ranging from 10^5 to 10^7, these expressions have, respectively, the form [1, 2]:

$$q_0(x) = 0.0296\,\lambda\,\frac{T_0 - T_{w0}}{x}\left(\frac{\rho V_0 x}{\mu}\right)^{0.8}\mathrm{Pr}^{0.43}, \tag{1}$$

$$\tau_0(x) = 0.0296\,\mu\,\frac{V_0}{x}\left(\frac{\rho V_0 x}{\mu}\right)^{0.8}, \tag{2}$$

$$\delta_0(x) = 0.37x\left(\frac{\rho V_0 x}{\mu}\right)^{-0.2} \tag{3}$$

In most cases of practical interest, however, the surface temperature is not a constant. Then, the heat flow calculated from formula (1), derived under the assumption of a constant surface temperature, can differ appreciably from the actual flow. Allowance for the variability of the surface temperature can be made, in a relatively simple way, with the aid of Seban's and Rubesin's formulas, obtained by applying the integral method to the analysis of the thermal boundary layer that develops within a dynamic layer [3, 4].

The literature lacks formulas for calculating shear stress for flows past a surface, a portion of which is in motion (for example, during the formation of a fluid film on the surface), while shear stress computations from formula (2) can lead in this case to results that are erroneous even with respect to the direction in which the friction forces are applied. It is, therefore, desirable to obtain an appropriate and, at the same time, simple method with which heat flows and shear stresses can be calculated for a variety of boundary conditions.

Definition of influence zones. It is proposed to extend formulas (1) and (2) to include stepwise-discontinuous conditions at the wall by replacing the characteristic values of the velocity V_0 and temperature T_0 at the boundary-layer boundary δ_0 by the corresponding velocity V_s and temperature T_s values at the boundary of the internal boundary layer that develops from the area s where the conditions at the wall begin to vary (see Figure 1). In this case, the linear dimension x should be replaced by x − s.

Similar to the approaches used by Rubesin and Seban, this approach is based on the physical prerequisite of the existence of such an internal layer that would concentrate in itself all the effects associated with the variability of the boundary conditions. In the present paper, the boundary of this layer is determined from formula (3) with allowance for the aforesaid changes in the characteristic values, i.e.,

$$\delta_s(x) = 0.37\,(x - s)\left[\frac{\rho V_s (x - s)}{\mu}\right]^{-0.2}. \tag{4}$$

This internal layer will be termed the "influence zone" of cross section s. In accordance with definition (4), the boundary layer with the boundary $\delta_0(x)$ is also the influence zone for s = 0.

We assume that the parameters of the gas beyond the influence zone (including the boundary of the zone) remain constant regardless of the nature of the changes in the boundary conditions at the wall. This condition makes it possible to determine the boundary of the influence zone and, correspondingly, the velocity and temperature values at this boundary for steady conditions at the wall. If the velocity and temperature distributions are assumed to obey the power law

$$\frac{V}{V_0} = \frac{T - T_{w0}}{T_0 - T_{w0}} = \left(\frac{\delta}{\delta_0}\right)^{1/7}, \tag{5}$$

by analyzing relations (4) and (5) simultaneously, we get

$$\frac{\delta_s}{\delta_0} = \left(1 - \frac{s}{x}\right)^{7/9},$$

$$\frac{V_s}{V_0} = \frac{T_s - T_{w0}}{T_0 - T_{w0}} = \left(1 - \frac{s}{x}\right)^{1/9}. \tag{6}$$

The relations obtained for V_s and T_s will be used in the solution of problems with variable conditions at the wall to be examined below as applications of the method proposed.

Heat transfer in the case of a variable wall temperature. Let us examine the flow past a plate of turbulent gas having a temperature T_0 and a velocity V_0. The temperature of the plate from its leading edge to cross section s is kept constant and equal to T_{w0}. Further downstream, the surface temperature of the plate changes abruptly to T_{ws}. For simplicity, and for clearer representation of the results obtained, we shall assume in the following that the changes in the characteristic physical parameters, which result from changes in the wall temperature, are negligible as compared with the absolute values of these parameters ($|T_{ws} - T_{w0}| \ll T_{w0}$).

According to the method proposed, expression (1) for the heat flow at x > s must be converted to the form

$$q_s(x) = 0.0296\,\lambda\,\frac{T_s - T_{ws}}{x - s}\left[\frac{\rho V_s (x - s)}{\mu}\right]^{0.8} \mathrm{Pr}^{0.43}. \tag{7}$$

Substituting the values for V_s and T_s from the relations (6) into relation (7) for the condition $T_{ws} = T_{w0}$, we get $q_s(x) \equiv q_0(x)$. Consequently, to calculate the heat flow for steady conditions at the wall, one may substitute into formula (1) an arbitrary linear dimension x − s and the corresponding values for the velocity and temperature at the boundary of the influence zone of section s. Although, strictly speaking, the "1/7 law" for the distributions of V and T is not applicable to a laminar sublayer, the fact that relation (7) holds for the case $T_{ws} = T_{w0}$ justifies to a certain extent the use of the heat transfer model proposed for values of $1 - (s/x) < 0.001$, which correspond to the region of the laminar sublayer.

For the condition $T_{ws} \neq T_{w0}$ in the problem under consideration, expression (7) yields

$$\frac{q_s(x)}{q_0(x)} = 1 - \frac{T_{ws} - T_{w0}}{T_0 - T_{w0}}\left(1 - \frac{s}{x}\right)^{-1/9}. \tag{8}$$

In the case where the wall temperature from x = 0 to x = s coincides with the gas temperature ($T_{w0} = T_0$), formula (8) takes the form

$$\frac{\mathrm{St}_s(x)}{\mathrm{St}_0(x)} = \left(1 - \frac{s}{x}\right)^{-1/9}, \tag{9}$$

where

$$\mathrm{St}_0(x) = q_0(x)/\rho c_p V_0 (T_0 - T_{w0});$$
$$\mathrm{St}_s(x) = q_s(x)/\rho c_p V_0 (T_0 - T_{ws}).$$

Formula (9) does not differ significantly from Seban's formula

$$\frac{\mathrm{St}_s(x)}{\mathrm{St}_0(x)} = \left[1 - \left(\frac{s}{x}\right)^{9/10}\right]^{-1/9}. \tag{10}$$

Figure 2 gives the results of calculations from formulas (9) and (10), and for comparison also experimental data [3] obtained at Reynolds numbers ranging

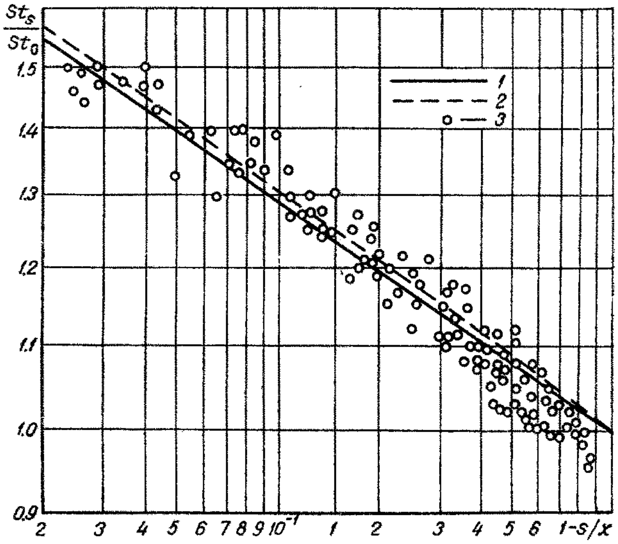

Fig. 2. Comparison of theoretical and experimental data for a stepwise varying surface temperature: 1) from formula (9); 2) from Seban's formula (10); 3) experimental data [3].

from $5 \cdot 10^5$ to $4 \cdot 10^6$. It can be seen that the relations (9) and (10) correlate well with the experimental data.

Comparing the results obtained from formulas (9) and (10), it becomes evident that the difference between them is maximum, and equal to 1.2% for $x \approx s$, while for x > s, it decreases monotonically. Since this difference lies within experimental uncertainty, it should be noted that formula (9) deserves preference over formula (10) inasmuch as it involves less computational labor.

In the case of an arbitrary wall temperature distribution for $x \geq s$, the results obtained for a stepwise varying temperatures can be generalized (as shown in [4]) by summing up the heat flux increments of all the area elements. Generalization of formula (8) to include the case of a continuously varying wall temperature leads to the expression

$$q_s(x) = \rho c_p V_0 \,\mathrm{St}_0(x) \times$$
$$\times \left[(T_0 - T_{w0}) - \int_s^x (1 - \xi/x)^{-1/9} (dT_w/d\xi)\, d\xi\right].$$

Shear stress at a moving surface. Let us examine a turbulent gas flowing at a velocity V_0 past a plate. The fairing of the plate between the leading edge and x = s is at rest, while further downstream it moves at a constant velocity V_{ws} in the same direction as the gas. In accordance with the method proposed, expression (2) for the shear stress at x > s should be transformed as

$$\tau_s(x) = 0.0296\,\mu\,\frac{V_s - V_{ws}}{x - s}\left[\frac{\rho (V_s - V_{ws})(x - s)}{\mu}\right]^{0.8}.$$

Substituting V_s from (6) into this formula, we get

$$\frac{\tau_s(x)}{\tau_0(x)} = 1 - \left(\frac{V_{ws}}{V_0}\right)^{1.8}\left(1 - \frac{s}{x}\right)^{-0.2}, \tag{11}$$

where $\tau_0(x)$ is the shear stress at the cross section under consideration for a surface in the state of rest. From an analysis of expression (11), it follows that, in the case of $V_0 > V_{ws}$, the ratio τ_s/τ_0 for $x \approx s$ is negative, i.e., that the friction forces applied at this point to the moving surface are directed opposite to the gas flow.

If formula (11) is generalized to include a continuously varying velocity of the surface, in the same manner as formula (8) was extended to include an arbitrarily varying wall temperature, one can obtain the following expression for the shear stress at a moving surface:

$$\tau_s(x) = \tau_0(x)\left[1 - \int_s^x (1 - \xi/x)^{-0.2}\, d(V_w/V_0)^{1.8}\right].$$

This expression and formula (11) still require experimental verification.

All the relations in this paper were obtained under the assumption that the velocity and temperature distributions in the boundary layer are governed by law (5) with an exponent of 1/7. Similar relations can be obtained also for an arbitrary exponent $1/n$ ($n \geq 1$). In this case, in accordance with [2], the exponents of the Reynolds numbers in the expressions (1)–(3) must be $(n + 1)/(n + 3)$, $(n + 1)/(n + 3)$, and $-(2/(n + 3))$, respectively. With the aid of operations similar to those performed with formulas (1)–(5), the expressions (6) for the boundary of the influence zone and for the velocity and temperature at this boundary take the form

$$\frac{\delta_s}{\delta_0} = \left(1 - \frac{s}{x}\right)^{\frac{n}{n+2}},$$

$$\frac{V_s}{V_0} = \frac{T_s - T_{w0}}{T_0 - T_{w0}} = \left(1 - \frac{s}{x}\right)^{\frac{1}{n+2}},$$

while expression (8) for the heat flow and expression (11) for the shear stress will transform to

$$\frac{q_s}{q_0} = 1 - \frac{T_{ws} - T_{w0}}{T_0 - T_{w0}}\left(1 - \frac{s}{x}\right)^{\frac{-1}{n+2}},$$

$$\frac{\tau_s}{\tau_0} = 1 - \left(\frac{V_{ws}}{V_0}\right)^{\frac{2n+4}{n+3}}\left(1 - \frac{s}{x}\right)^{\frac{-2}{n+3}}.$$

NOTATION

V is the velocity; T is the temperature; ρ is the density; c_p is the specific heat; μ and λ are the viscosity and heat conduction coefficients, respectively; q is the heat flow; τ is the shear stress; δ is the boundary layer thickness; x is the distance from the plate leading edge to the cross section under consideration; s is the length of the initial portion of the plate with stable conditions at the wall; St is the dimensionless heat-transfer coefficient (Stanton number); subscript 0 refers to parameters in the case of stable conditions at the wall; subscripts s refer to parameters in the case where a boundary layer forms at cross section s.

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